SOL HW 3.5

May 16, 2018 3:28 PM

Name:______________________________ Date:____________________________

Section 3.5 Cosecant Secant and Cotangent Functions

1. Determine each value to 3 decimal places:

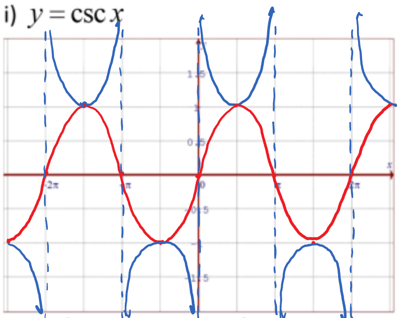
a) $\csc 110°$ $= -0.364$	b) $\cot 130°$ -0.839	c) $\sec 95°$ $= -11.474$	d) $\csc 64°$ $= 1.113$
e) $\cot 233°$ $= 0.754$	f) $\sec 100°$ -5.759	g) $\cot 45°$ 1.000	h) $\sec 112°$ $= -2.669$

2. Determine the exact value of each of the following without a calculator

a) $\csc 45°$ $= \frac{1}{\sin 45} = \sqrt{2}$	b) $\cot 180°$ $= \frac{1}{\tan 180°} = \infty$	c) $\sec 60°$ $\frac{1}{\cos 60°} = 2$	d) $\csc 135°$ $\frac{1}{\sin 135°} = \sqrt{2}$
e) $\cot \frac{\pi}{3} = \frac{1}{\tan \frac{\pi}{3}} = \frac{2}{\sqrt{3}}$	f) $\sec \frac{2\pi}{3} = \frac{1}{\cos \frac{2\pi}{3}}$ $= -2$	g) $\cot 225°$ $\frac{1}{\tan 225} = 1$	h) $\csc 300°$ $\frac{1}{\sin 300°} = \frac{-2}{\sqrt{3}}$
i) $\sec \frac{11\pi}{6}$ $\frac{1}{\cos \frac{11\pi}{6}} = \frac{2}{\sqrt{3}}$	j) $\cot \frac{4\pi}{3}$ $\frac{1}{\tan(\frac{4\pi}{3})} = \frac{1}{\sqrt{3}}$	k) $\csc \pi$ $\frac{1}{\sin \pi} = \infty$	L) $\sec \frac{5\pi}{6}$ $\frac{1}{\cos(\frac{5\pi}{6})} = \frac{1}{(-\frac{\sqrt{3}}{2})} = \frac{-2}{\sqrt{3}}$

3. Graph the following function for $-2\pi \le \theta \le 2\pi$. Indicate the Period, Amplitude, Domain, and Range:

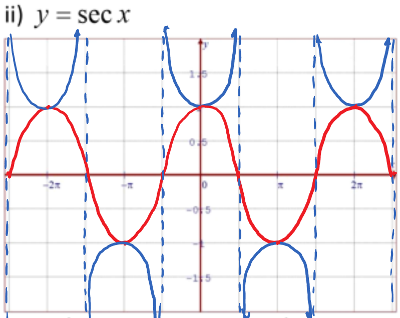
i) $y = \csc x$



Period: 2π Amplitude: None

Domain: $x \in \mathbb{R}; x \neq n\pi$ Range: $y \le -1$ or $y \ge 1$

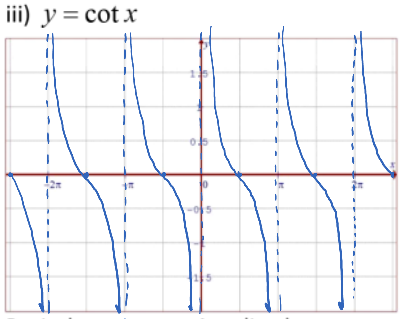
ii) $y = \sec x$



Period: 2π Amplitude: None

Domain: $x \in \mathbb{R}$, $x \neq \frac{(2n+1)\pi}{2}$ Range: $y \le -1$ or $y \ge 1$

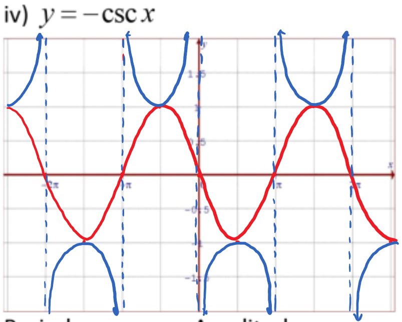
iii) $y = \cot x$



Period: π Amplitude: None

Domain: $x \in \mathbb{R}$, $x \neq n\pi$ Range: $y \in \mathbb{R}$

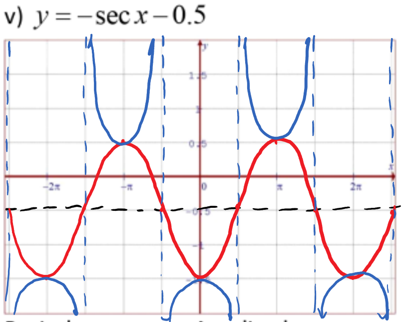
iv) $y = -\csc x$



Period: Amplitude:

Domain: Range:

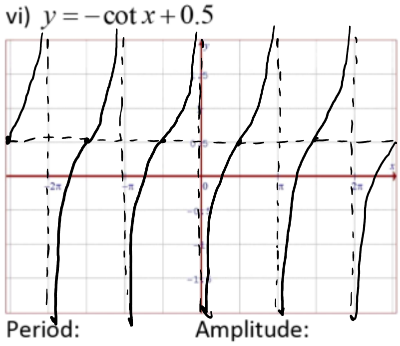
v) $y = -\sec x - 0.5$



Period: Amplitude:

Domain: Range: $y \ge 0.5$ or $y \le -1.5$

vi) $y = -\cot x + 0.5$



Period: Amplitude:

Domain: Range:

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14. Simplify the following in terms of "sine" and "cosine" only:

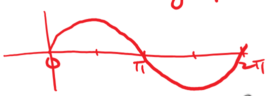
a) $(\sec x\csc x-\cot x)(\sin x-\csc x)$ $c^2+s^2=1$ $s^2-1=-c^2$ $\left(\frac{1}{\cos x}\left(\frac{1}{\sin x}\right)-\frac{\cos x}{\sin x}\right)\left(\sin x-\frac{1}{\sin x}\right)$ $\left(\frac{1}{\sin x\cos x}-\frac{\cos^2 x}{\sin x\cos x}\right)\left(\frac{\sin^2 x-1}{\sin x}\right)$ $\left(\frac{1-\cos^2 x}{\sin x\cos x}\right)\left(\frac{\sin^2 x-1}{\sin x}\right)$ $\left(\frac{\sin^{\cancel{2}} x}{\cancel{\sin x}\cos x}\right)\left(\frac{-\cos^2 x}{\cancel{\sin x}}\right)=-\cos x.$	b) $\dfrac{\frac{\cot x+1}{\cot x}-1}{\frac{\cot x-1}{\cot x}-1}=\dfrac{\frac{\cot x}{\cot x}+\frac{1}{\cot x}-1}{\frac{\cot x}{\cot x}-\frac{1}{\cot x}-1}$ $=\dfrac{\frac{1}{\cot x}}{-\frac{1}{\cot x}}=-1$
c) $\cot x+\tan x$ $\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}$ $\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$ $\frac{1}{\sin x\cos x}.$	d) $\dfrac{\csc^2 x+\sec^2 x}{\csc x\sec x}$ $\frac{[\csc^2 x+\sec^2 x]}{(\csc x)(\sec x)}\cdot\frac{(\sin^2 x\cos^2 x)}{(\sin^2 x\cos^2 x)}$ $\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$ $=\frac{1}{\sin x\cos x}$
e) $\sec A\sqrt{\dfrac{1-\sin^2 B\sin^2 A}{1+\cos^2 A\tan^2 B}}$ ($\tan^2 A$) $\frac{\left[1-\sin^2 B\left(\frac{\sin^2 A}{\cos^2 A}\right)\right]\cos^2 A\cos^2 B}{\left[1+\cos^2 A\left(\frac{\sin^2 B}{\cos^2 B}\right)\right]\cos^2 A\cos^2 B}$	f) $\dfrac{\sec x}{\tan x+\cot x}$ $\frac{\left(\frac{1}{\cos x}\right)(\sin x\cos x)}{\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)(\sin x\cos x)}$ $=\frac{\sin x}{\sin^2 x+\cos^2 x}=\sin x$

4. Indicate the general formula for the vertical asymptotes of $y=\cot x$

① $\cot x=\frac{\cos x}{\sin x}$ ← Look at denominator to find asymptotes

② $\sin x=0$

$x=\sin^{-1}(0)$



∴ $x\neq n(\pi)$; $n=\pm 0,1,2,3,...$

5. Given each expression, calculate the value of θ for $0<\theta<2\pi$

a) $\csc\theta=-2$ $\frac{1}{\sin\theta}=-2$ $\sin\theta=-\frac{1}{2}$ $\theta=\sin^{-1}(-\frac{1}{2})$ $\theta=210°, 330°$ or $\frac{7\pi}{6},\frac{11\pi}{6}$	b) $\sec\theta=\dfrac{2\sqrt{3}}{3}$ $\frac{1}{\cos\theta}=\frac{2\sqrt{3}}{3}$ $\cos\theta=\frac{3}{2\sqrt{3}}$ $\cos\theta=\frac{\sqrt{3}}{2}$ $\theta=\frac{\pi}{6},\frac{11\pi}{6}$	c) $\cot^2\theta=1$ $\frac{1}{\tan^2\theta}=1$ $\tan^2\theta=1$ $\tan\theta=\pm 1$ $\therefore\theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$	d) $3\csc\theta=-2\sqrt{3}$ $\frac{3}{\sin\theta}=-2\sqrt{3}$ $\frac{3}{-2\sqrt{3}}=\sin\theta$ $-\frac{\sqrt{3}}{2}=\sin\theta$ $\theta=\frac{4\pi}{3},\frac{5\pi}{3}$
e) $\cot\theta=-\sqrt{3}$ $\frac{1}{\tan\theta}=-\sqrt{3}$ $\tan\theta=-\frac{1}{\sqrt{3}}$ $\theta=\frac{5\pi}{6},\frac{11\pi}{6}$	f) $\sec\theta=\infty$ $\frac{1}{\cos\theta}=\infty$ $\cos\theta=0$ $\theta=90°, 270°$ • The x-int is equal to zero when the terminal arm is pointing up at 90° or down at 270°. $\theta=\frac{\pi}{2},\frac{3\pi}{2}$	g) $\cot\theta=0$ $\frac{1}{\tan\theta}=0$ $\tan\theta=\infty$ $\theta=90°, 270°$ $\tan\theta$ is undefined when $\cos\theta=0$. b/c $\tan\theta=\frac{\sin\theta}{\cos\theta}$ ← zero $\theta=\frac{\pi}{2},\frac{3\pi}{2}.$	h) $\csc\theta=-1$ $\frac{1}{\sin\theta}=-1$ $\sin\theta=-1.$ $\theta=\frac{3\pi}{2}$ The y-coord of the terminal arm is −1 only when it is pointing down at 270°.

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i) $6\sec\theta = 4\sqrt{3}$ $\sec\theta = \frac{4\sqrt{3}}{6}$ $\frac{1}{\cos\theta} = \frac{2\sqrt{3}}{3}$ $\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$	j) $3\csc\theta + 2\sqrt{3} = 0$ $3\csc\theta = -2\sqrt{3}$ $\csc\theta = -\frac{2\sqrt{3}}{3}$ $\sin\theta = -\frac{\sqrt{3}}{2}$ Q3 Q4 $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$	k) $\tan\theta - \sqrt{3} = 0$ $\tan\theta = \sqrt{3}$ $\theta = \tan^{-1}(\sqrt{3})$ Q1 $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$	L) $\sqrt{3}\sec\theta + 2 = 0$ $\sec\theta = -\frac{2}{\sqrt{3}}$ $\cos\theta = -\frac{\sqrt{3}}{2}$ Q2 $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$ Q3
m) $2\csc^2\theta - 1 = 3$ $2\csc^2\theta = 4$ $\csc^2\theta = 2$ $\sin^2\theta = \frac{1}{2}$ $\sin\theta = \pm\frac{1}{\sqrt{2}}$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	n) $\tan^2\theta = 9$ $\tan\theta = \pm 3$ $\theta = \tan^{-1}(\pm 3)$ $\theta_1 = 1.249r$ Q1 $\theta_2 = 1.8925r$ Q2 $\theta_3 = 4.3906r$ Q3 $\theta_4 = 5.0341r$ Q4	o) $\cos^2\theta - \cos\theta - 2 = 0$ $A^2 - A - 2 = 0$ $(A-2)(A+1) = 0$ $A = 2$ $A = -1$ $\cos\theta = 2$ $\cos\theta = -1$ No soln $\theta = \pi$	p) $\frac{-2}{\csc\theta} + \csc\theta = 1$ $-2 + \csc^2\theta = \csc\theta$ $\csc^2\theta - \csc\theta - 2 = 0$ $\csc\theta = 2$ $\csc\theta = -1$ $\sin\theta = \frac{1}{2}$ $\sin\theta = -1$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = \frac{3\pi}{2}$

6. Given that $\sin^2\theta + \cos^2\theta = 1$ and $\tan^2\theta = 1.25$, what is the value of $\sec^2\theta$?

$$\frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{1}{c^2}$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1.25 + 1 = \sec^2\theta$$
$$2.25 = \sec^2\theta$$

7. If θ is an angle whose measure is not an integer multiple of $90°$, prove that $\cot\theta - \cot 2\theta = \frac{1}{\sin 2\theta}$

$= \cot\theta - \cot 2\theta$

$= \frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin 2\theta}$

$= \frac{\cos\theta}{\sin\theta} - \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta - \cos^2\theta + \sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{2\sin\theta\cos\theta} = \frac{1}{\sin 2\theta}$

Note: $\sin 2\theta = \sin(\theta + \theta)$
$= \sin\theta\cos\theta + \sin\theta\cos\theta$
$= 2\sin\theta\cos\theta$.

8. Simplify the following expression: $\frac{\sec A - \cos A}{\tan A}$

$\frac{\frac{1}{\cos\theta} - \cos\theta \quad \times\cos\theta}{\frac{\sin\theta}{\cos\theta} \quad \times\cos\theta}$

$= \frac{1 - \cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta.$

$\boxed{\text{a) } \sin A}$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

9. Simplify the following expression: $\frac{1 + \sec A}{\tan A + \sin A}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

$= \frac{\left(1 + \frac{1}{\cos\theta}\right)\cos\theta}{\left(\frac{\sin\theta}{\cos\theta} + \sin\theta\right)\cos\theta}$

$= \frac{\cos\theta + 1}{\sin\theta + \sin\theta\cos\theta}$

$= \frac{\cos\theta + 1}{\sin\theta(1 + \cos\theta)} = \frac{1}{\sin\theta} = \csc\theta.$

10. Simplify the following expression: $\frac{\tan A + \cot A}{\sec A}$

$\frac{\left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]\cos\theta \times \sin\theta}{\left[\frac{1}{\cos\theta}\right]\cos\theta \times \sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} = \csc\theta$

a) $\sin A$ b) $\cos A$ c) $\sec A$ $\boxed{\text{d) } \csc A}$ e) $\cot A$

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11. Simplify the following expression: $\dfrac{1+\sin A}{1+\csc A}$

$\dfrac{(1+\sin A)\sin A}{\left(1+\frac{1}{\sin A}\right)\sin A}$

$\dfrac{(1+\sin A)\sin A}{(\sin A+1)}$

$= \sin A$ //

$\boxed{\text{a) } \sin A}$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

12. Simplify the following expression: $\dfrac{1+\sec A}{1+\cos A}$

$\dfrac{\left(1+\frac{1}{\cos A}\right)\cos A}{(1+\cos A)\cos A}$

$\dfrac{(\cos A+1)}{(1+\cos A)\cos A}$

$= \dfrac{1}{\cos A}$

a) $\sin A$ b) $\cos A$ $\boxed{\text{c) } \sec A}$ $= \sec A$ // d) $\csc A$ e) $\cot A$

13. Simplify the following expression: $\dfrac{1+\cot A}{1+\tan A}$

$\dfrac{(1+\cot A)\tan A}{(1+\tan A)\tan A}$

$= \dfrac{(\tan A+1)}{(1+\tan A)\tan A}$

$= \dfrac{1}{\tan A}$

$= \cot A$ //

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ $\boxed{\text{e) } \cot A}$

14. Prove that both sides of the equation are equal: $\dfrac{\sin A+\cos A\cot A}{\cos A\csc A} = \sec A$

$\dfrac{\left(\sin A + \cos A \frac{\cos A}{\sin A}\right) \times \sin A}{\left(\cos A \times \frac{1}{\sin A}\right) \times \sin A} = \sec A$

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A} = \dfrac{1}{\cos A} = \sec A$

15. Prove that both sides of the equation are equal: $\dfrac{1-\cos A}{\sin A} = \dfrac{1}{\csc A+\cot A}$

$\dfrac{1-\cos A}{\sin A} = \dfrac{1}{\left(\frac{1}{\sin A}\right)+\left(\frac{\cos A}{\sin A}\right)}$

$= \dfrac{1}{\left(\frac{1+\cos A}{\sin A}\right)}$

$= \dfrac{\sin A}{1+\cos A} \cdot \dfrac{(1-\cos A)}{(1-\cos A)}$

$= \dfrac{\sin A\,(1-\cos A)}{1-\cos^2 A}$

$= \dfrac{\cancel{\sin A}\,(1-\cos A)}{\sin^{\cancel{2}} A}$

$= \dfrac{1-\cos A}{\sin A}$ //

16. Challenge: What are all the values of "x" between 0 *and* 2π that satisfy the equation?

$\left(5+2\sqrt{6}\right)^{\sin x} + \left(5-2\sqrt{6}\right)^{\sin x} = 2\sqrt{3}$

Hint #1 $(5+2\sqrt{6})(5-2\sqrt{6})$
$= 25 - 24$
$= 1$ //
$\therefore 5+2\sqrt{6} = \dfrac{1}{5-2\sqrt{6}}$ //

Hint #2
$(\sqrt{3}+\sqrt{2})^2 = (\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})$
$= 3 + \sqrt{6} + \sqrt{6} + 2$
$= 5 + 2\sqrt{6}$. //

Hint #3 $(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})$
$= 5 - 2\sqrt{6}$
$= \dfrac{1}{5+2\sqrt{6}}$
$\therefore (\sqrt{3}-\sqrt{2})^{-2} = 5+2\sqrt{6}$ //

$A^{\sin x} + \dfrac{1}{A^{\sin x}} = 2\sqrt{3}$

$B + \dfrac{1}{B} = 2\sqrt{3}$

$B^2 + 1 = 2\sqrt{3}(B)$

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$B^2 - 2\sqrt{3}B + 1 = 0$

$B = \dfrac{2\sqrt{3} \pm \sqrt{12-4}}{2}$

$= \dfrac{2\sqrt{3} \pm 2\sqrt{2}}{2}$

$B = \sqrt{3} \pm \sqrt{2}$

$\overline{\qquad\quad}$
$\quad 2$

$B = \sqrt{3} \pm \sqrt{2}$

$\therefore (5+2\sqrt{6})^{\sin x} = \sqrt{3}+\sqrt{2}$. or $(5+2\sqrt{6})^{\sin x} = \sqrt{3}-\sqrt{2}$.

$(\sqrt{3}+\sqrt{2})^{2\sin x} = (\sqrt{3}+\sqrt{2})^{1}$

$(\sqrt{3}-\sqrt{2})^{-2\sin x} = (\sqrt{3}-\sqrt{2})^{1}$

$\therefore 2\sin x = 1$

$\sin x = \frac{1}{2}$

$\underline{\underline{x = 30°, 120°.}}$

$\therefore -2\sin x = 1$

$\sin x = \frac{-1}{2}$

$\underline{\underline{x = 210°, 330°}}$